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Simplified stochastic modeling and simulation of unidirectional fiber reinforced composites

Guido Camata[a,1], Ross Corotis[a,*], Enrico Spacone[b,c,2]

[a]*Department of CEAE, University of Colorado, Boulder, CO 80309-0428, USA*
[b]*Department of PRICOS, Facolta di Architettura, University G. D Annunzio of Chieti-Pescara, viale Pindaro 42, Pescara 65127, Italy*
[c]*Department of Civil, Environment and Architectural Engineering, University of Colorado, Boulder, CO 80309-0428, USA*

Abstract

Stochastic prediction of the failure strength of composite materials has been the subject of research for the last four decades. These studies have generally focused on expensive materials with high quality control for use in aerospace applications. For these applications it has been economically feasible to obtain reliable statistical data. To extend this research to the analysis of composites with widespread use in civil engineering, the complicated models developed in the past need to be simplified. This paper proposes a simplified stochastic two-dimensional model to predict the strength distribution of single-ply unidirectional composites. The model is studied via Monte Carlo simulation and accounts for the following parameters: specimen size, fiber strength distribution, fiber–matrix properties and load transfer at broken fibers. The classical load sharing rules to model the load redistribution at broken fibers are modified to consider different material properties. A parametric study investigating the parameters that affect the strength and the failure pattern of the composite is presented. The numerical results are consistent with experimental data analyzed. The study is used to quantify the effect of various material and geometric characteristics on resultant performance and failure properties.

Keywords: Fiber reinforced polymer composite materials; Monte Carlo simulation; Loading sharing rule; Weibull distribution; Glass fiber reinforced polymers

1. Introduction

Fiber reinforced polymers (FRPs), have been widely used in recent years for the rehabilitation of existing structures in the form of seismic retrofit, service load strengthening, and damage repairs. Their use to date in new structural systems has been limited to few demonstration projects. A broad design base, such as that available for many metals, has not been yet compiled.

A composite is a material system consisting of two or more phases, where mechanical performance and properties are designed to be superior to each individual material acting independently. Due to the heterogeneous properties of the constituents and the manufacturing process these materials show a high degree of variability. Besides, testing of full-scale specimens it is not economically feasible and hence much of the experimental research has been carried out using small-scale models. The strength of composites is strongly influenced by the volume size and therefore design of large structures from data collected using small-scale tests is complex.

For these reasons, a deterministic approach, such as the well-known rule of mixture, cannot lead to satisfactory results. To have confidence in this approach requires information on the statistics nature and reliable models. It is easy to understand why the stochastic prediction of the failure strength of composite materials has been the subject of research for the last four decades.

To develop improved failure models, it is important to take into account the stochastic nature of the composites. This paper focused on the behavior of single-ply unidirectional composites. The proposed model is studied using Monte Carlo simulation. In the simulations, the strength of the single fiber is assumed to follow a Weibull distribution, and the stress redistribution is modeled using a conceptually new loading sharing rule.

Several models have been proposed. Rosen [9] assumed that the distribution of the fiber's strength was governed by a Weibull distribution and carried out an experimental study to validate his model. Zweben [11] improved Rosen's

* Corresponding author. Tel.: +1-303-735-0539; fax: +1-303-492-7317.
E-mail addresses: corotis@colorado.edu (R. Corotis), guido.camata@colorado.edu (G. Camata), espacone@unich.it (E. Spacone).
[1] Fax: +1-303-492-7317.
[2] Tel.: +39-085-453-7276; fax: +39-085-453-7255.

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model considering the effect of stress concentration. These two models have been widely used and modified since their publication and they can be considered as the starting point of sophisticated models developed in the 1980s and 1990s.

2. Weakest link theory

The reinforcing fibers are brittle, and therefore can be assumed to behave linearly up to failure. Fibers contain flaws, which can propagate easily across the fiber. The stress at which the flaw extends depends upon the size of the flaw, and the resultant crack determines the point of failure in any individual length of fiber. The chain of bundle proposed by Rosen model describes mathematically this behavior and has proven to be reliable to simulate single fiber strength [9]. This theory is based on the assumption that the length of the fiber material is made up of smaller elements linked together and that failure of the material as a whole occurs when any one of these elements fail. The probability of failure of each link subjected to a stress which increases from zero to σ is described by the distribution function $P(\sigma)$ [3,6]. It assumed that $P(\sigma)$ describes the strength distribution for any element and that each $P_f(\sigma)$ is an independent randomly-distributed variable for each element along the fiber.

The probability of failure of a chain of n elements is given by:

$$P_f(\sigma) = 1 - [1 - P(\sigma)]^n \quad (1)$$

$P_f(\sigma)$ can be described by:

$$P_f(\sigma) = 1 - e^{-\varphi(\sigma)} \quad (2)$$

This approach follows the approach of Weibull. The probability of failure of n elements can be simplified as follow:

$$P_f(\sigma) = 1 - e^{-(L/L_0)(\sigma/\sigma_0)^m} \quad (3)$$

where L_0 is a reference length, s_0 is a scale parameter at length L_0 and m is the shape parameter of the Weibull distribution.

3. Model description

Past experimental tests showed that when an individual fiber of a composite fails, the composite as a whole does not fail due to a redistribution of the load among the other fibers. Therefore, to apply the weakest link theory to composite materials each element of the individual fiber chain is considered to be part of an element composed of many parallel fibers.

The composite can be described as an idealized series-parallel system as in Fig. 1.

A basic mechanics assumption is that the fibers carry only tensile stresses and the matrix only shear stresses.

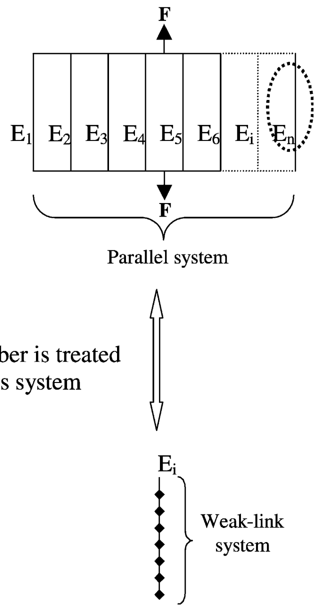

Fig. 1. Idealized series-parallel model.

When a fiber fails or there is debonding along the matrix–fiber interface, the fiber is assumed to slip with respect to the matrix with a sliding resistance τ acting across the fiber–matrix interface. The interfacial shear allows the transfer of tensile stress from the fiber break point into the matrix and then back into the fiber at points farther along the fiber [2]. For a constant τ, the stress increases linearly from zero at the break to the full applied stress, as shown in Fig. 2 where σ_{app} is the stress applied. The stress drops to zero linearly as we approach $z = z_0$ [1]:

$$P_i(z) = \sigma_{app} - 2\tau(z - z_0)/r \quad (4)$$

where τ is the shear stress carried by the matrix, z_0 is the point at which failure occurs and r is the radius of the $\{i\}$ fiber. The stress dropped due to a fiber break at z_0 must be transferred to the remaining unbroken fibers and this is the key to the damage accumulation problem.

A configuration of broken fibers $\{i\}$ all located in a single plane z gives rise to stress transfer factor K_{im} acting on the remaining unbroken fibers in the same plane. Several loading sharing models have been proposed. In the global loading sharing (GLS) model, the stresses are transferred equally to all remaining intact fibers, independent of the relative fiber positions in the plane [3]. The stress on a particular unbroken fiber m in plane z is given by

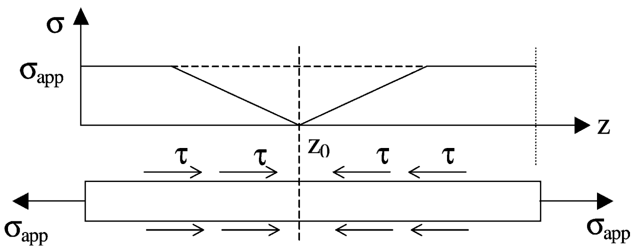

Fig. 2. Shear stress induced by rupture of a fiber or by interface debonding.

the following equation:

$$\sigma_m(z) = \sigma_{\text{app}} + \sum_l K_{im} p_i(z) \tag{5}$$

in which $p_i(z)$ is defined in Eq. (4).

In the case of the GLS model the stress transfer factor is given by the following:

$$K_{im} = 1/(n_f - n_b) \tag{6}$$

where n_f is the number of fibers in the composite and n_b is the number of broken or slipping fibers at cross-section z.

A more realistic treatment of load transfer was introduced by Zweben and Rosen [9,11] and further developed by Hedgenpeth and Van Dyke [4,5]. They proposed that the load previously taken by the broken fiber is now transferred via the matrix only to the adjacent fibers at the plane of the break, around the break and then back to the original fiber [2]. In the case of a local loading sharing (LLS) model the stress transfer factor $K_{m,n}$ was calculated by Hedgenpeth and Van Dyke for a two- and three-dimensional problem [4,5]. In the three-dimensional case, they showed that the in-plane stress transfer on fibers (m,n) due to a single break at origin (0,0) under stress σ can be expressed as $K_{m,n}\sigma$, with:

$$K_{m,n} = -q_{m,n}/q_{0,0} \tag{7}$$

$$q_{m,n} = -\frac{2}{\pi}\int_0^{\pi}\int_0^{\pi}\cos(m\phi)\cos(n\vartheta)\sqrt{\sin^2\beta - \sin^2\gamma}\,d\phi\,d\vartheta \tag{8}$$

where:

$$\gamma = (\pi - \phi)/2 \text{ and } \sin^2\beta = 1 + \sin^2\left(\frac{\phi}{2}\right) \tag{9}$$

Eq. (8) is evaluated using numerical integration. It should be noted that as $m, n \to \infty$, $q_{m,n} \to 1$. When $m = 15$ and $n = 0$, the stress transfer factor assumes a value which is the 0.33% of that of the fiber adjacent to the fiber broken, and thus can be considered negligible.

In the proposed model, the stress is transferred from the broken fiber to the 30 adjacent fibers through the LLS model.

Noting that $\left(\sum_{m=1}^{15} K_{m,0}\right)\sigma < \sigma$ for a composite made up of more than 30 fibers, it is possible from Eqs. (7) to (9) to calculate the stress loss due to incomplete stress redistribution. For a composite with 1000 fibers the result is:

$$\left(1 - 2\sum_{m=1}^{15} K_{m,0}\right)\sigma = 0.032\sigma \tag{10}$$

Therefore, each time a fiber breaks or slips, the 1000 fiber system through the LLS model loses 3.2% of the stress dropped by the broken fiber. This loss of energy accumulates every time a fiber breaks. To avoid this unrealistic dissipation, the stress not distributed through the LLS model is redistributed using a GLS model as shown in Fig. 3.

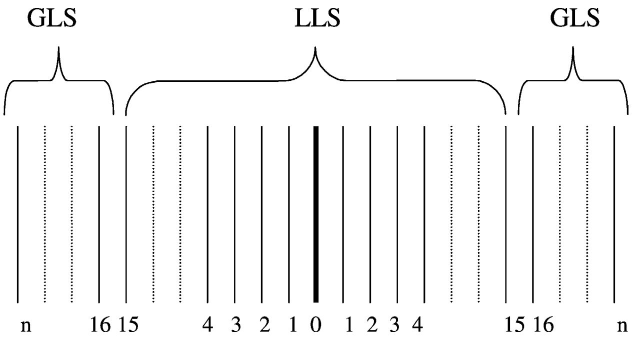

Fig. 3. Schematic of the sharing rule used in the model.

In our model, the results of Hedgenpeth and Van Dyke [5] (which is represented by Material 1) is used as an upper bound to calculate $K_{m,n}$ and the GLS model as a lower bound, as depicted in Fig. 4.

In the model the shape parameter m is approximated using the coefficient of variation (COV) of the data set [1]:

$$m = 1.2/\text{COV} \tag{11}$$

Using different values of K it is possible to model varying interface shear strength and therefore different constituent materials.

3.1. Stress concentration at the free edge

High stresses can arise at the free edge of the laminate and it can lead to premature failure of the composite. Various reasons can induce this increase in stress. For a unidirectional composite the effect can arise from the fact that the fibers are broken when the specimen is cut [10]. This happens because the fibers at the free edge are not perfectly straight. Moreover, if a fiber slips or fails at the boundary of the laminate it can transfer load trough the LLS model only to part of the composite. For example, if fiber n on the left edge of the array in Fig. 3 fails it can transfer stresses only to the right. To take into account these issues the loading sharing model was modified at the free edge of the laminate, introducing a more severe load redistribution (higher stress to adjacent fibers).

4. Simulation description

The model is studied using a Monte Carlo simulation technique. For each experiment, 10,000 simulations of different bundle sizes were carried out.

1. The simulation starts with the generation of Weibull-distributed strengths given appropriate scale parameters.
2. Then a force is applied as in a controlled tensile test until the stress reaches the strength σ_1 of the weakest fiber in the bundle.
3. The stress in that fiber is set to zero, and the fiber stress is redistributed in accordance with the chosen sharing rule.

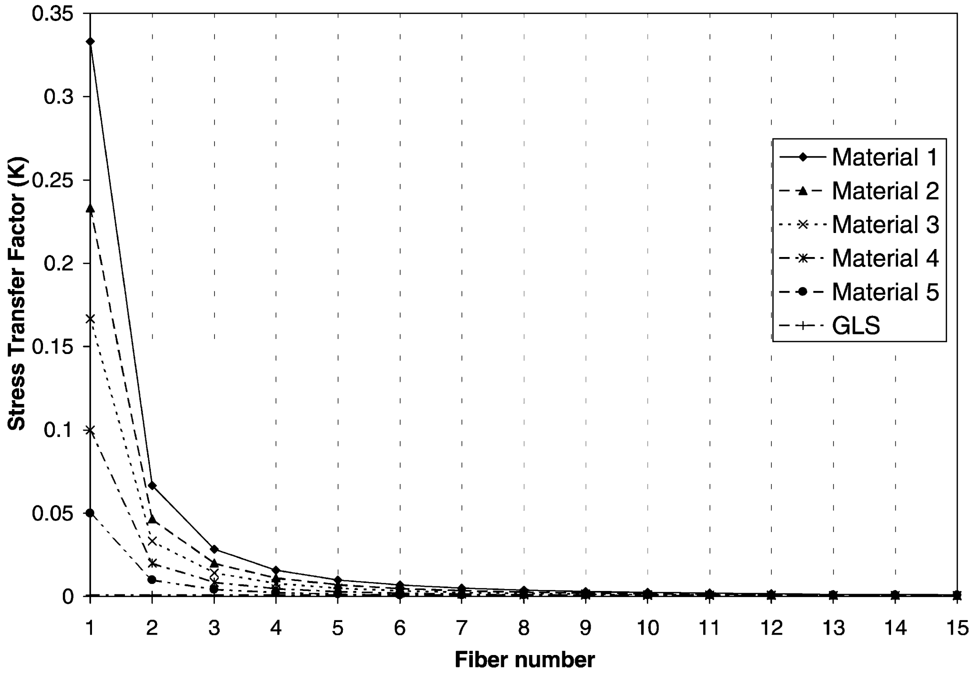

Fig. 4. stress transfer factors used in the model.

4. Keeping the applied load constant, step 3 is repeated until no further breaks occur.
5. If the system can support the applied load following step 4, the load is then increased until the critical stress is reached in the fiber with the smallest residual strength. Steps 3 and 4 are then repeated.
6. The composite is defined as broken when no further increase in load is needed to break all the fibers of the system (the system can not support the applied load).
7. The fracture strength of step 6 is the strength of the dry fiber bundle σ_{bundle}.

The tensile strength of the composite is found applying the following formula:

$$\sigma_{\text{FRP}} = f\sigma_{\text{bundle}} + (1 - f)\sigma_{\text{matrix}} \quad (12)$$

where f is the fiber area fraction and σ_{matrix} is the matrix tensile strength.

5. Results

5.1. Experiments analysis

The model is applied to study the experiments reported by Rosen [9]. The specimens consisted of a single layer of parallel glass fibers glued together by two type of epoxy resin, epoxy B and C. Epoxy yield strengths were, respectively, $\sigma_B = 75$ MPa and $\sigma_C = 34$ MPa. The specimens had length $l = 25.4$ mm (1 in.), width $w = 12.7$ mm (0.5 in.) and thickness $t = 0.1$ mm (0.06 in.). The fiber

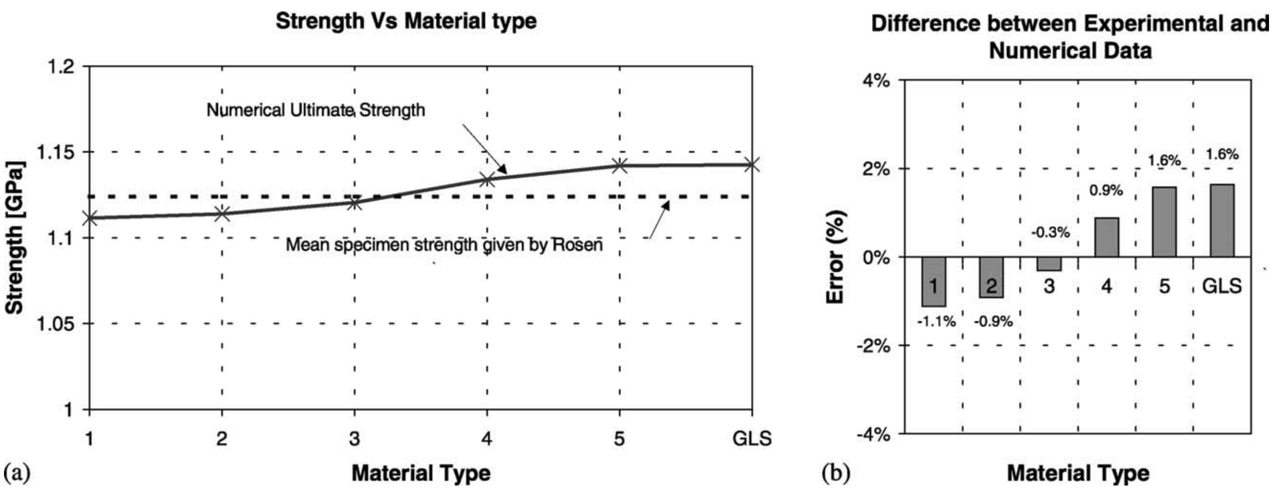

Fig. 5. Comparison between experimental and numerical results for type B specimens.

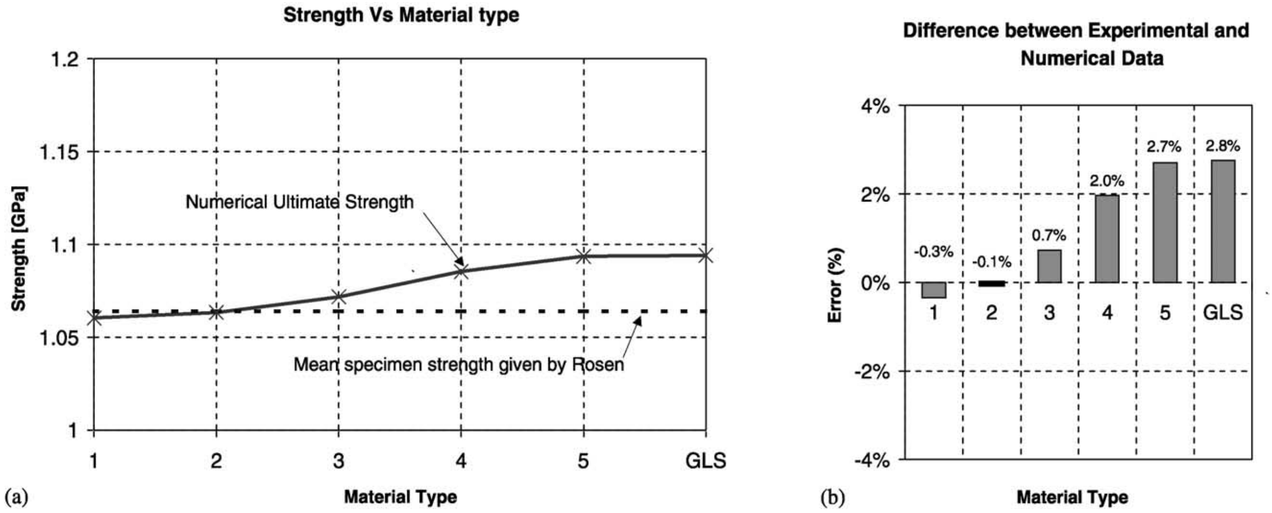

Fig. 6. Comparison between experimental and numerical results for type C specimens.

area (and volume) fraction f was 50%. The fibers had cross-section radius $r = 0.044$ mm (0.0264 in). The number of fibers for the specimens embedded in epoxy B ranged between $n_B = 126$ and 130, and for the specimens prepared with epoxy C, $n_c = 124-128$. The COV of the fiber strength for specimens type B was $COV_B = 0.0441$ and for type C was $COV_C = 0.0466$. The effective shape parameters for the composite were calculated using Eq. (11) and were computed to be $m_B = 27.21$ and $m_C = 25.75$. The scale parameter was $\sigma_0 = 1.250$ Gpa, at a gauge length of $l_0 = 25.4$ mm [8].

The simulations were carried out using different interfacial shear strengths as defined by material types in Fig. 4, Fig. 5 presents results for epoxy B.

The numerical results agree very well with the experimental data, with Material 3 being in closest agreement to Rosen's test composite. The simulations were accomplished for Type C specimens as well and for these set of experiments the simulations using Material 2 gave the smallest error (-0.1%) as shown in Fig. 6.

A group of two or more adjacent broken fibers is defined as a cluster. When the number of adjacent broken fibers grows to be large enough the cluster will tend to become unstable and cause the failure of the composite. Such a group is defined as the critical cluster. Figs. 7 and 8 show the average cluster size and the average number of clusters formed at ultimate load in the composite by the simulations for both specimen types. The figures do not consider the simulations in which no clusters formed.

Table 1 shows the percentage of simulations in which at least one cluster formed in type C specimen. As expected, there is an increased tendency of forming clusters for higher interfacial shear strength.

5.2. Parametric study

In the parametric study a dry fiber bundle of 128 fibers is modeled and the load sharing model used is Material 3. In the first test, the scale parameter was $\sigma_0 = 1.250$ Gpa, with a gauge length of $l_0 = 25.4$ mm. The COV of the fiber

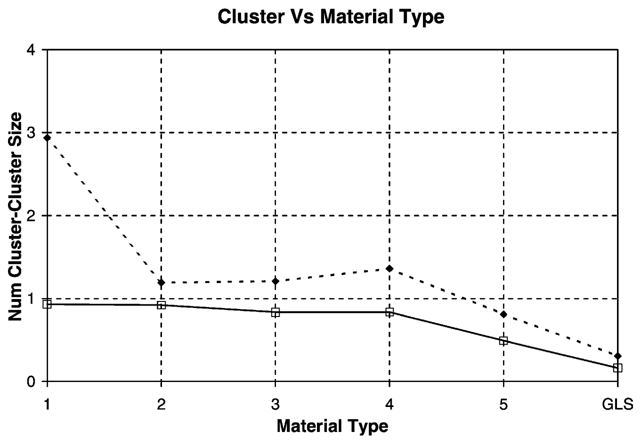

Fig. 7. Cluster size and number of cluster at ultimate for epoxy B.

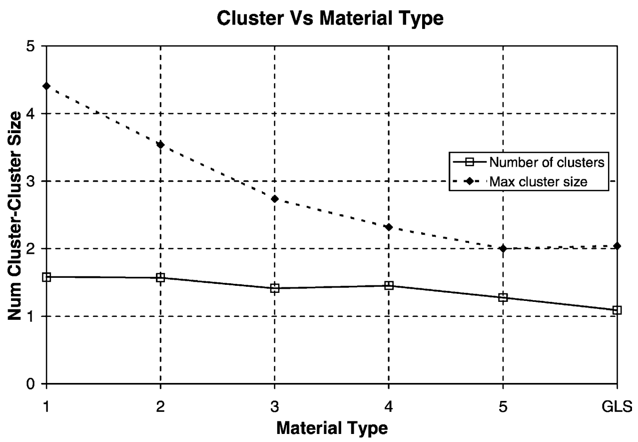

Fig. 8. Cluster size and number of cluster at ultimate for Epoxy C.

Table 1
Ductility of the material in function of the interfacial shear strength adopted

Material type	Percentage
1	62.2
2	61.6
3	61.5
4	59.1
5	41.0
GLS	15.4

bundle was varied between 0.0441 and 0.25 as shown in Figs. 9 and 10.

As depicted in Fig. 9(a) increasing the fiber strength variability decreases the load at which the weakest fiber fails or slips as well as the composite strength. Along with this decrease there is a less brittle behavior, as shown in Fig. 9(b). Fig. 10(a) shows that the number of clusters and the critical cluster size increase with COV. Therefore, for fibers with high variability and high shear fiber/interface strength, the failure of the composite tends to be localized.

Consequently also the probability of cluster formation increases for materials with high variability, as reflected in Fig. 10(a).

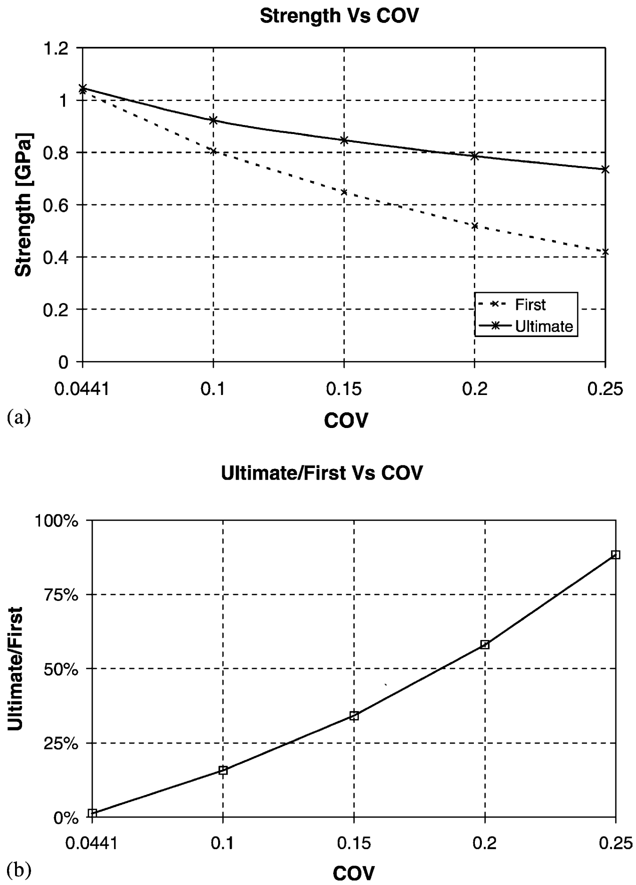

Fig. 9. (a) and (b) Strength in function of the COV.

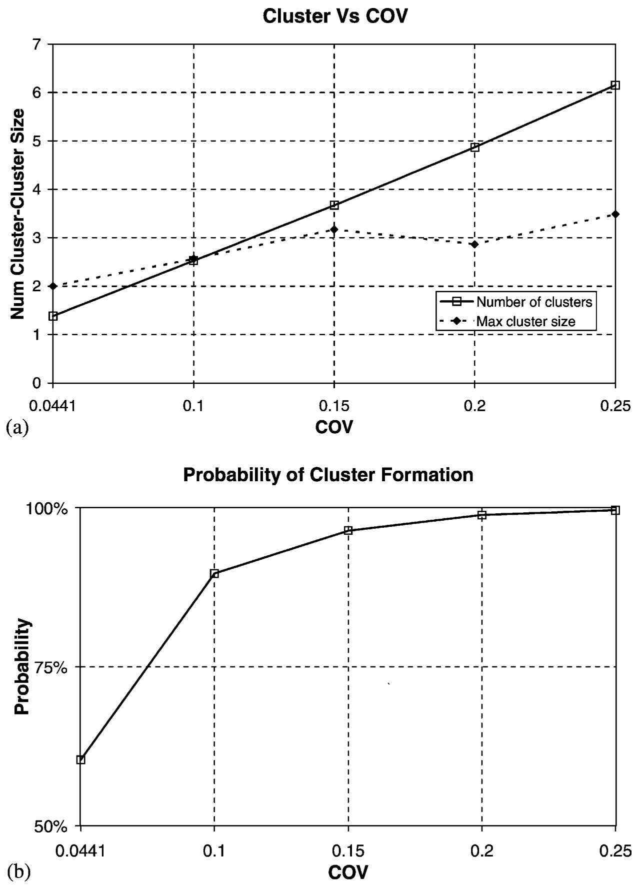

Fig. 10. (a) Cluster size and number of cluster at ultimate. (b) Probability of cluster formation.

In the second study, the COV was kept constant at 0.15 and $\sigma_0 = 1.250$ Gpa. To simulate the size effect, L of Eq. (3) was considered as variable. Fig. 11 shows that the dry bundle strength decreases when increasing the length of the fibers, as expected.

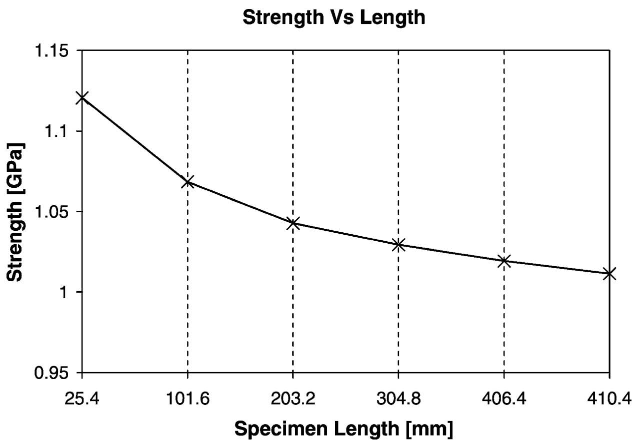

Fig. 11. Size effect.

Curtin [1] reported that inferring the strength distribution using Eq. (3) begins to be inaccurate for length larger than ten times the gauge length.

6. Discussion and conclusions

The agreement between the simulations and the experiments is very good, and the parametric analysis has addressed several important issues concerning the failure process of single-ply unidirectional composites. The model proposed simplifies the physical background using a two-dimensional array of fiber. Past researches show that this disposition underestimates the predicted strength since the load sharing is less advantageous than for a three-dimensional arrangement [7]. The Rosen specimens analyzed in this paper contain a single layer of glass fiber, which facilitates the use of a two-dimensional model to compare with actual experimental results. The good agreement forms a basis to extend the current model to three dimensions. Attention was put into modifying the existing models to avoid any energy loss due to stress transfer each time a fiber broke or slipped. In fact, using a pure LLS model does not transfer all the stress dropped by a broken fiber, and the GLS model does not describe realistically the nature of a laminate. The combined model proposed allows, as for GLS, preservation of the Principle of Energy Conservation, while using a more realistic load sharing concept.

In recent years, Monte Carlo simulations have given important insight into the failure process of composite materials [8]. They revealed that the composite could fail into two different modes, for dispersed failure in a 'ductile-like' behavior and for localized failure related to a brittle-like failure. Several factors influence these failure modes, among them the interface shear strength between the fibers and the matrix and the variability in strength of the fibers. In dispersed failure, the failure surface is brush-like and tends to be associated with weak interface strength and large variability in strength. Figs. 12 and 13 show that a weak interface between fiber and matrix increases the pseudo-ductility of the composite and is associated with a ductile-like failure. Figs. 7 and 8 illustrate that weakening of the matrix–fiber interface reduces the likelihood of cluster formation, and the fracture occurs because of dispersed failure. The same failure mode was obtained combining a strong interface with a large variability in the fibers, as shown in Fig. 10. These results agree with past experimental studies [8].

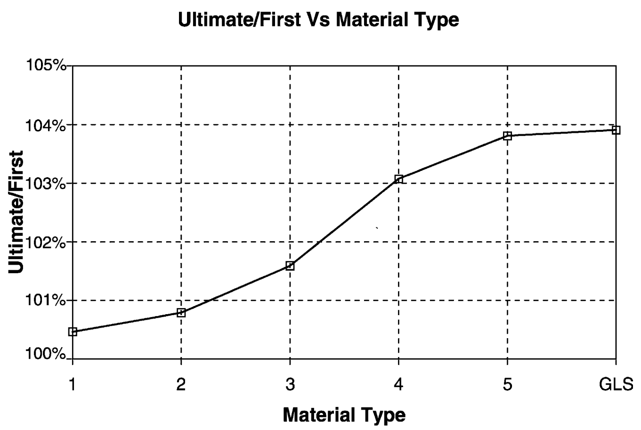

Fig. 12. Ratio between ultimate and first load for type B specimens.

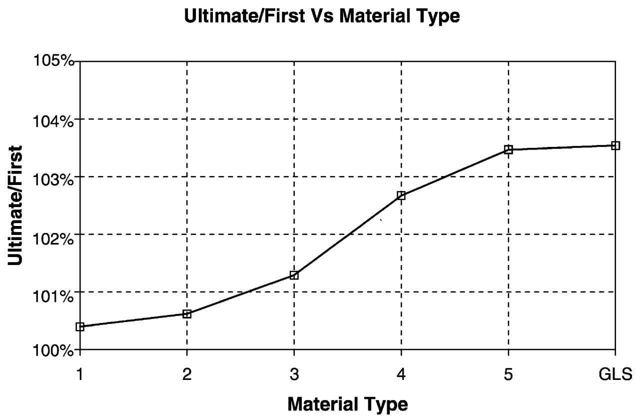

Fig. 13. Ratio between ultimate and first load for type C specimens.

Fig. 8 indicates that for Material 3, which showed the best fit to the experimental results, the critical cluster size is equal to 2.74. This is in remarkable agreement with Phoenix [8] who found analytically that the critical cluster size for the same set of experiments was equal to three.

Because of the assumption made considering the two-dimensional arrangement of the fiber bundle, it is not possible with this model to capture exactly the failure process of the material. A planar array arrangement tends to form a localized failure and the simulations confirmed this behavior. Actual laminates have a three-dimensional disposition, and a planar disposition underestimates the strength of this material.

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